**Time Value of Money**

**Future Value /Compounding Techniques**

1. Single amount or Lumpsum
2. Annuity
3. Doubling Period
4. Effective interest rate/Effective Rate of Interest
5. Continuous compounding

**Present Value /Discounting Techniques**

1. Single Amount or Lumpsum
2. Series of Cash flow/Uneven Cash flows
3. Annuity
4. Growing Annuity
5. Perpetuity
6. Perpetuity with Growth

**Equated annual instalment**

**Loan Amortisation Schedule**

Future Value calculations

Suppose Rs 100000/- is deposited in a fixed deposit scheme of a bank for 3 year period, on which the bank offers an of 10% per annum. What will be the amount receivable at the maturity period of 3 years?



1st Method individual year

FV=PV(1+i)1=

1st year FV=100000(1+.10)1

FV =110000

2nd year FV=110000(1+.10)1

FV=121000

3rd year FV=121000(1+.10)1

FV=133100

2nd Method formula method

FV=100000(1+.10)3

FV =133100

3rd Method using the table

FV=100000(1.3310)

FV=133100

Calculate Future value of Rs 100000 at the rate of Interest of 15% p.a for a period of 3 years

FV=100000(1+.15)3

15FV=152087.5

FV=100000(1.5209)

Re.1 becomes 1.5209

1000000 becomes ?1,52090

FV=PV(1+i)n

=1(1.15)1

1.15

1(1+.15)2

1.3225

1(1.15)3

1.520875

Calculate the future value of Rs 50000 at the rate of 12%pa for 4 years.

FV=50000(1+.12)4

Continuous Compounding

Interest is credited for more than once in a year

Half yearly=2times

Quarter yearly=4times

Monthly =12times

Weekly= 52 times

Daily=365 times

Suppose Rs 1000000 was deposited for a period of 2years in a bank which is offering 10% interest p.a payable half yearly. What will be the future value of the investment?

1st method

1st half year =1000000x(10/100)x(6/12)=50000=1050000

2nd half year=1050000x.10x.5=52500=1102500

3rd half year=1102500x.10x.5=55125=1157625

4th half year=1157625x.10x.5=57881.25=12,15,506.2

2nd Method Formula

FV=PV(1+i/f)nXf

FV=1000000(1+.10/2)2x2

FV=12,15,506.25

3rd Method table

FV=PV(FVIFr/f,nxf)

FV=1000000(1.2155)

FV=1215500

Annual interest =10%

Half yearly interest= 10%/2=5%

3.Calculate the future value of Rs 100000 at the rate of 10% for 1 year, if interest is compounded quarterly?-Ans.Rs.110381

4.Rs 1000 today is equivalent to how much at the end of 2 years when interest is calculated half yearly? The rate of interest is 20%.

Ans.1461.4

5.X has invested Rs 240000 at annual rate of interest of 10% . what is the amount after 3 years if the compounding is done :

1 Annually Ans 319440

2 Semi Annually Ans 321622.95

3 Quarter yearly Ans 322773.31

4 Monthly Ans 323563.64

A company offers a fixed deposit scheme whereby Rs 20000 matures to Rs 25250 after 2 years , on a half yearly compounding basis . If the company wishes to amend the scheme by compounding interest every quarterly what will be the revised maturity value?

Interest not given

FV=25250

PV=20000

N=2years

F=2 times

25250=20000(1+i/2)2x2

25250 ={1+(i/2)}4

20000

1.2625={1+(i/2)}4

Using the table refer the 4th period row and find where you get the value 1.2625 =6%

As 6% is half yearly hence multiply by two to get the annual interest =12%

Interest 12%

FV when interest is compounded quarterly=

FV= 20000(1+[.12/4])2x4= 25334

How many years will it take for an initial investment of Rs 10000 to grow to Rs 67275 , if it is invested at 10% int Compounded annually? 20 years

At what rate of interest Rs 10000 be compounded annually for it to grow to Rs 45950 in 16 year period? 10% interest

P ltd. Had revenues of Rs 10000 in 1990 which increased to Rs 100000 in the year 2000. What was the compound growth rate in the revenues? 26%

**Effective Interest Rate or Effective Rate of Interest**

Effective Rate of Interest =[1+(i/f)f]-1

What is the effective rate of Interest, if the rate of interest is 12% per annum, when compounded annually, semiannually, quarterly and monthly ?

Annual

ERI=[1+(i/f)] f -1

ERI=[1=.12/1]1 -1

ERI=12%

Semi annual

ERI=[1+(.12/2)] 2 -1

ERI=0.1236 x100=12.36%

Quarterly

ERI=[1+(.12/4)] 4 -1=12.55%

Monthly

ERI=[1+(.12/12)] 12 -1=12.68%

Weekly

ERI=[1+(.12/52)] 52 -1=12.73%

Suppose a 1year deposit of Rs.1,00,000/- interest is calculated half yearly at an interest rate of 10%p.a. ,if it calculated quarterly at the same rate. What will be the future value of the deposit and the effective rate of interest?

Continuous compounding on daily basis

FV=PV ein

e=2.7183

i x n

Banks may pay interest continuously

If the company is compounding the interest on every day basis what will be value for Rs 100 at the rate of interest 12% for 2 years period?

FV=100x(2.7183).12x2 =127

FV for a single sum

FV continuous, EIR

FV of Annuity

**Discounting or finding Present Value**

Discounting refers to finding the present value of cash flows or out flows happening on a future date .It refers to finding today’s value of future cash flows.

Discounting involves the following:

Finding present value of a single cash flow

Finding present value of a series of cash flow

**Single Cash flow**

1. Finding present value of one time cash flow happening on a future date
2. What is the present value of Rs 10,00,000 receivable after 3 years i=10%? PV=1000000(0.7513)=751315
3. Find the present value of Rs 200000 payable after 2 years 10%. PV=200000x0.8264=165280

FV=PV(1+i)n

PV= FV( table value )

When there is multiple discounting

PV= FV

[1+(i/f) nxf]

4.Suppose a debtor owes Rs 100000after 2 years but intends to pay today. What is amount he has pay today at the rate of 10 % interest?

The amount he needs to pay is Rs 82644.63 and not Rs 100000. Since the amount of Rs 82644.63 can be invested today for 2years at 10% pa , which amounts to Rs 10000 by the end of 2 years

5.Find the present value of Rs 100000 received at the end of 2 years at a discount rate of 10%?=

PV=100000(0.8264)=82640

Calculate the present value of the following:

Rs 2000 to be received at the end of 2 years @ 10% compounded annually PV= 2000 =1652.8

(1+.1)2

7.Rs 5000 to be received at the end of 1 year at 9% compounded quarterly

PV= 5000 =4574.56

(1+.09/4)1 x4

8.Rs 1000 to be rece

ived at the end of 6months at 8% compounded monthly

FV=1000

N=.5 years

I=8%

F=12months

PV= 1000

1+(.08/12)0.5x12

PV= 1000

(1.0067)6

PV= 1000

1.0408

PV=Rs 961.5

8.Rs 4000 to be received at the end of year 1 at 12% compounded monthly

PV= 4000

[1+(.12/12)1x12 ]

PV=3549.6

**Series of Cash flow**

Suppose a firm has estimated the following receipts by the end of each year for the next 3 years, what will be the present value of the cash flows at 12% interest?

|  |  |  |  |
| --- | --- | --- | --- |
| Year | Cash receipts/Flow | PVIF @12 | Cash flows XPVIF |
| 1 | 100000 | 0.8929 | 89290 |
| 2 | 300000 | 0.7972 | 239160 |
| 3 | 200000 | 0.7118 | 142360 |
|  |  |  | 470810 |

PV=CF1+ CF2 +CF3

(1+i)1  (1+i)2 (1+i)3

PV=100000 +300000 +200000 =

(1+.12)1 (1+.12)2 (1+.12)3

PV=89285.7+239158.16+142356.04=470799.9

1.An investor receives Rs1000, Rs 1500,Rs 800,Rs 1100 & Rs 400 respectively at the end of 1 through 5 years.Find the present value of this stream of uneven cash flows if the interest rate is 8%.

2.Determine the present value of cash inflows at the rate of interest 14%

|  |  |  |  |
| --- | --- | --- | --- |
| Year | CF |  |  |
| 1 | 3000 |  |  |
| 2 | 2500 |  |  |
| 3 | 2100 |  |  |
| 4 | 7000 |  |  |
| 5 | 4000 |  |  |
| 6 | 6000 |  |  |

3.What is the present value of the following

|  |  |  |  |
| --- | --- | --- | --- |
| Year | CF |  |  |
| 5 | 5000 |  |  |
| 2 | 6000 |  |  |
| 3 | 8000 |  |  |
| 4 | 9000 |  |  |
| 5 | 8000 |  |  |

4.What is the present value of Rs 100000 receivables after 60 years from now , if it is discounted at 10%.

Future value

1. Rs.50000 at 16% after 9 years.
2. Rs.5000 at 11% after 3 years.
3. Rs.15000 at 11% after3 years.
4. A company is offered a contract which has the following terms . An cash outlay of Rs 15000 followed by a cash inflows of Rs 17900 after 3 years . what is company’s rate of return on this contract.

**Annuity**

An annuity is a series of equal payment made at fixed intervals for a specific number of periods.

**Ordinary Annuity / Growing Annuity**

Payments occurring at the end of each period

Example Payments on loans, mortgages etc.

**Annuity due**

If payments are made at the beginning of each period.

Example Rental payments, life insurance premium etc.

Annuity Due

FV=(Annuity x FVAFr,n)(1+r)

PV=(AnnuityxPVARr,n)(1+r)

1.A 4year annuity of Rs 1000 each is invested at the beginning of the year at the rate of interest of 10%. Calculate the PV of the Annuity.

2.A recurring deposit of Rs 100 is made in the beginning of each of 4 years starting now at 6% per annum. What will be the total deposit at the end of 4 years?

FV=(100x4.3746)(1+.06)=Rs.463.7

Future Value of an Annuity or Terminal Value of Annuity



Present Value of an Annuity



Present Value of Growing Annuity

X will receive Rs 100000 by the end of first year from an investment scheme and the revenue is expected to grow by 10% per annum each year upto fifth year. What is the maximum amount he can consider investing in a scheme, if he is expecting a return of 15%p.a?

PV of Growing annuity=

100000 [1-(1.1)5/(1.15)5]

(0.15-0.10)

=Rs 398583.79

Suppose you have a right to harvest a teak plantation for the next 6 years over which you expect to get 100000cubic feet of teak per year. The price per cubic feet of teak is Rs 500, but it is expected increase at a rate of 8% per year. The discount rate is 15%. What is the present value of teak that you can harvest from the teak forest?

500x100000=5,00,00,000

PV of Growing Annuity=

5,00,00,000[1-(1.08)6/(1.15)6]

(0.15-0.08)

=22,42,50,313.5

Future value of Annuity

1. Rs.1000 for 3 years at 10% interest



FVof Annuity =Annuity(FVAF r,n)

FV of Annuity =1000(3.3100)=Rs.3310

1. Rs 10000 for 10 years at 9%,10% and 11% interest.

FV of Annuity @ 9% n10years=10000(15.193)Rs.151930

FV of Annuity @ 10%=Rs 159370

FV of Annuity @11%=Rs .167220

1. A 4 year Annuity of Rs 3000 per year is deposited in a bank account that pays 9% interest compounded yearly. What is the future value of the Annuity?

FV of Annuity =3000(4.5731)=Rs.13719.3

**Present Value of Annuity**

1.A 12 payment annuity of Rs 10000 is invested for 12years and earns a interest of 10%.What will the present value of the Annuity? Rs 68137/-

2. A Project fetches an annual return of Rs 250000 for 5 years. If the cost of money is 15% calculate the present value of the project.

|  |  |  |  |
| --- | --- | --- | --- |
| Years | Cash flows | PVIF @ 15% | CFxPVIF=  PVof CF |
| 1 | 250000 | 0.8696 | 217400 |
| 2 | 250000 | 0.7561 | 189025 |
| 3 | 250000 | 0.6575 | 164375 |
| 4 | 250000 | 0.5718 | 142950 |
| 5 | 250000 | 0.4972 | 124300 |
|  | 1250000 |  | 838050 |

**Present Value of Perpetuity**

Perpetuity refers to cash flows of same amount in regular intervals for ever .That is , when same amount is received or paid in regular intervals forever, it is called as perpetuity.

Example:Pension ,Interest paid for irredeemable bonds

PV of Perpetuity =Cash flows

r

1.What is the present value of Cashflow of

Rs 60 per year forever, at an interest rate of 8%?

PV=Rs 750

2.Find the present value of Rs 1000 received per year forever at a discount rate of 10%.

PV=Rs 10000

Present Value of Growing Perpetuity

PVof Growing Perpetuity= P1

(r-g)

3.Calculate the present value from the following information:

Cash flow at the end of year 1= Rs 10000

Growth rate =8%

Period of Cash Flow =infinity

Discount rate =12%

PV= 10000

(.12-.08)

PV=Rs.250000

**Doubling Period**

Rule 72

Rule 69

If the Interest rate is 14% ,what are the doubling period as per Rule 72 and Rule 69.

Rule 72=72/14=5.14years

Rule 69=5.27 years= 0.35+(69/14)

**Equated Annual instalments**

When a loan is borrowed or advanced, repayments happen in equated instalments . Or payment of each instalment boots payment towards interest and repayment towards principal calculation of equated

instalment is based on the concept of time value of money.

Equated Installment =

Loan Amount

PV of Annuity of Re1 at i for n years

The above formula can be used in case of annual payment of installments where the instalments are to be paid more than once a year that is half yearly, quarterly, monthly etc the formula for calculation of equated annual installment must be

LoanAmount

PV of Annuity of Re1 at i/f for nxf years

1.A loan of Rs 10,00,000 is borrowed for a period of 5 years at an interest of 10% pa.If the repayments are made at the end of each year in equated installments, what is the amount of installments? You are required to prepare a Loan Amortisation Schedule .

Equated Annual Instalment=

10,00,000

(PVAF 10%,5years)

10,00,000

3.7908

Rs 2,63,796.56

Half yearly

LoanAmount

PV of Annuity of Re1 at i/f for nxf years

1000000

(PVAF(10%/2),5x2)

1000000

(PVAF(5%),10)

1000000

(7.7217)

Equated half yearly129505.16

1000000

(PVAF(10%/12),5x12)

**Loan Amortization Schedule**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Year** | **Loan Amount**  **Outstanding at the beginning of the year**  **1** | **Interest at10%in Rs**  **2** | **Total Amount outstanding by the end of the year in Rs**  **1+2=3** | **Equated**  **Annual Instalment s**  **4** | **Loan Amount outstanding at the end of the year**  **3-4=5** |
| **1** | 1000000 | 100000 | 1100000 | 263796.56 | 836203 |
| **2** | 836203 | 83620 | 919823 | 263796.56 | 656026 |
| **3** | 656026 | 65603 | 721629 | 263796.56 | 457832 |
| **4** | 457832 | 45783 | 503615 | 263796.56 | 239818 |
| **5** | 239818 | 23982 | 263800 | 263796.56 | - |

**Sinking Fund**

Sinking fund Is a fund which is created out of fixed payments each period to accumulate to a future sum after a specific period. Companies generally create sinking fund to retire bonds on maturity.

Suppose a firm wants to get ₹21,875 at the end of 4 years from now. How much should it deposit each year at an interest rate of 6%, so that it grows to ₹21875 at the end of the 4th year.

FV=A(FVAFr,n)

A=FV/(FVAFr,n)

A=FV(1/FVAF r,n)

A= 21875(1/4.375)=5000

A company has issued debentures of ₹50,00,000 to be repaid after 7 years. How much should the company invest in sinking fund earning12% interest in order to repay debentures after 7 years.

A=5000000/10.089=4,95,589

Capital recovery factor.

Capital Recovery Factor is the annuity of an investment for a specified time at a given rate of interest. It helps in the preparation of a loan amortization or loan repayment schedule.

Suppose you have borrowed a 3 year loan of ₹10,000 at 9% interest from your employer to buy a motorcycle. If your employer requires 3 equal end of year repayments, then what should be your annual instalments?

PV=A(PVAFr,n)

10000=A/(2.531)

A=3951

Sadhu Lal is borrowing ₹50,000 to buy a low-income group house if he pays equal instalments for 25 years and 4% interest on outstanding balance. What is the amount of instalment? What is the amount of instalment if quarterly payments are required to be made?